

# Bimodality in management quantities and additional growth data in the assessment for snow crab in the Eastern Bering Sea

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## Introduction

Estimates of management quantities from the 2015 assessment for snow crab in the Eastern Bering Sea were bimodal. That is, when fitting the model with different starting values for estimated parameters, different ‘solutions’ were converged upon. These solutions had gradients less than 0.01 and produced a variance-covariance matrix, indicating they were stable local minima. This bimodality was shown to be a product of the use of piece-wise linear growth models in which a change-point was estimated that determined at what size the growth model switched from one linear function describing the relationship between pre-molt length and post-molt length to another. One of the key problems with this model formulation was that data were not available over the region in which the change point was estimated to occur. Consequently, the precise location of the change point was uncertain.

Poorly-behaved likelihood surfaces are a known problem for some size-structured assessment models. Until 2016, issues of instability in the likelihood surface (and the resulting bimodality in management quantities) were addressed in the snow crab assessment by ‘jittering’ the starting values for estimated parameters (i.e. choosing different random values within the bounds of each estimated parameter), running the model many times after different random jitters, then choosing the model that had the lowest likelihood. However, in 2016 the stable local minima had likelihoods such that none appeared to be significantly better than the other. Consequently, there was no defensible way to choose between the modes of management quantities using maximum likelihood methodologies.

The proposed (and adopted) solution to bimodal management quantities with similar likelihoods was to use Bayesian methods to allow the uncertainty around the modes in management quantities to be incorporated into management advice. This appeared to address the problem by producing an OFL that was lower than the highest mode estimated via maximum likelihood methods and higher than the lowest mode estimated via ML methods. Bayesian methodologies presented an attractive stop-gap in representing model uncertainty by propagating uncertainty forward into the management quantities by producing posterior distributions of the management quantities. These distributions also provide an avenue to base the buffers applied to the total allowable catches on a quantitative measure of uncertainty in the data, rather than an arbitrary number. However, Bayesian methodologies are time consuming to implement (running Markov Chain Monte Carlo algorithms can take several days), determining whether or not a model has ‘converged’ is difficult, and specifying priors can be a contentious endeavor that can influence the outcome of the assessment.

During 2017, a model was proposed (and adopted) that estimated mature female natural mortality. When this model was ‘jittered’, it no longer showed the bimodality in management quantities. Furthermore, additional growth data became available in late 2017, after the assessment was presented. These data span the gap in the growth data in which a change point is estimated. Given this background, there are two objectives for this document. First, I evaluate the impact of adding the new growth data into the model on the stability and magnitude of estimates of quantities used in management. Second, I explore the differences and merits of Bayesian methodologies when compared to maximum likelihood frameworks.

## Methods

The results from four models are presented here.

1. 2016\_oldgrowth:

- Accepted model for 2016 with slight changes introduced in 2017
- Survey data before 1982 dropped
- Split survey selectivity period in 1987
- Estimate survey availability parameters for BSFRF survey in logit space with a penalty
- Uses growth data from 2016 and 2017 assessments (i.e. growth relationships are fit to 18 and 22 observations of pre/post molt lengths for females and males, respectively)
- Natural mortality for mature females is fixed
- Uses catch and survey data through 2017 (i.e. is comparable with the 2017 models below)

2. 2016\_newgrowth:

- Identical to 2016\_oldgrowth, except the additional growth data provided after the 2017 assessment season are incorporated into the model. The additional data include 25 and 45 new pre/post molt observations for males and females, respectively.

3. 2017\_oldgrowth:

- Accepted model for 2017
- Uses growth data from 2016 and 2017 assessments
- Natural mortality for mature females is estimated

4. 2017\_newgrowth:

- Identical to 2017\_oldgrowth, but the additional growth data are used

Models were all jittered 100 times then runs that did not have a gradient less than 0.002 were removed. MCMC was performed for model 2017\_newgrowth, which entailed running 10 million iterations, saving every 5000th, then thinning the resulting chain to eliminate auto-correlation. Model 2017\_newgrowth was also altered so that management quantities (e.g. the OFL, B35, F35, FOFL) were declared as 'sdreport' variables, which allowed their standard deviations to be calculated. This was accomplished by adding a chunk of code to the procedure section that executed the functions that calculate management quantities when it is the 'sd\_phase'.

```
if(sd_phase())
{
  Find_F35();
  Find_OFL();
}
```

Parameter estimates, contributions to the objective function of each likelihood component, calculated management quantities, jittered output, and models fits to all data sources are included below.

## Results

The jittering analysis demonstrated the bimodality of the 2016 accepted model (2016\_olddata), but the remaining models did not present bimodality in management quantities (Figure 1 & Figure 2). This shows that the bimodality in management quantities can be removed either by adding the new growth data for females or by estimating mature female natural mortality (M). This observation raises the question of which ‘fix’ for bimodality should be adopted: using the new growth data, estimating mature female M, or both.

### Model fits and population processes

Models that estimated mature female M fit the data much better overall as seen through the objective function value (Table 3). Large improvements to the fits to the female growth data (Figure 3) and the mature survey biomass (Figure 4) were a key reason for this change. Estimating mature female M improved the fits to the female mature biomass at the time of the survey by reducing the estimated biomasses. If mature female M was not estimated, a broken-stick model was still estimated to fit the female growth data. Presumably, this was a result of the model needing to ‘kill off’ the biomass resulting from larger sized individuals more quickly than the specified natural mortality allows in order to fit the data, so estimated growth was slower for larger individuals when natural mortality was fixed for mature females. These improvements to the likelihood came even with a large increase in the penalty associated with the prior on M (Table 3). Estimating mature female M also restored the expected relationship between female and male natural mortality—female M is expected to be higher than male (Table 2). However, it also decreased in models where mature female M was not estimated, M for mature females was specified as 0.23 per year, mature male M was estimated as 0.25 per year, immature M (for both sexes) was estimated as 0.43 per year. In models where mature female M was estimated, M for mature females was estimated as 0.36 per year, mature male M was estimated as 0.27, immature M (for both sexes) was estimated as 0.28.

In addition to changing estimated growth for females, estimating M for mature females had relatively large impacts on survey selectivity (Figure 5). Survey catchability moved from  $\sim 0.5$  for females in the second survey era (1989-present) to 1 when estimating M for mature females. Catchability for males moved only slightly in the second survey era when estimating mature female M, but smaller crabs were more selected. Estimated catchability increased in the first survey era (1982-1989) for both sexes when estimating mature female M. An estimated survey catchability of 1 for females was somewhat concerning, given BSFRF experimental data that suggest otherwise. From a modeling perspective, it is perhaps unsurprising, though. The model seemed to ‘want’ to correct for some of the observed fluctuations in female biomass by using fishing mortality—increasing catchability magnifies the influence of whatever fishing mortality was occurring on females. Couple this with a relatively small contribution of the BSFRF data to the objective function and it is perhaps unsurprising that estimated catchability for females is estimated as high as it is.

Retained catch data were fit by all models well, with little discernible differences among models (Figure 6). Female discard data were fit adequately given the specified uncertainty (Figure 6 & Table 3). Male discard data during the period for which data exist (early 1990s to the present) were well fit by every model with little discernible difference (Figure 6). Fits to the trawl data were adequate for all models given the uncertainty in the data (Figure 6).

Retained catch size composition data were fit well by all models (Figure 7); trawl size composition data were generally well fit in most years. All models performed similarly in fitting the trawl size composition data (Figure 8 & Table 3). Fits to survey size composition data were not very different among scenarios (to the eye; Figure 9 & Figure 10), but 2017\_oldgrowth fit the data better according to the likelihoods (Table 3).

Small differences in the probability of maturing at size existed among models. For males, models in which M for mature females was estimated produced slightly higher probabilities of maturing for crab in the 50-100mm range than models that did not estimate M for mature females (Figure 11). The pattern was somewhat more complicated for females because both the estimation of M for mature females and the change in the growth curve as a result of additional growth data influence the estimated probability of maturing. The largest changes in the probability of maturing for females among models occurred in the 50-70mm range and the

model in which the additional growth data were used and mature female M was estimated produced the highest probabilities of maturing over this range. Other size classes of females had very similar estimated probabilities of maturing across models.

Small changes in estimates of directed fishing mortality existed among the models in the early years of the estimated time series, but recent estimates were similar for all models (Figure 12). Fishing mortality associated with bycatch from the trawl fisheries was estimated to be higher when mature female M was estimated, and these changes in trawl F were accompanied by estimated trawl selectivity that was slightly shifted to larger sizes. Estimated female discard mortality increased by nearly 50% when estimating mature female M, but was still very small compared to other sources of fishing mortality (e.g. 0.003 vs average directed F of  $\sim 0.8$ ).

Fits to the biomass estimates from the BSFRF selectivity experiments were nearly identical for all models (Figure 4). Slight differences in fits to the length composition data for the BSFRF selectivity experiments existed between models (Figure 13)—models in which mature female M was estimated fit the data better (Table 3) than those that did not. Large differences existed among model estimates of availability and selectivity for the BSFRF selectivity experiments (Figure 14).

Estimated patterns in recruitment were very similar for all models (Figure 15). Models in which mature female M was estimated produced recruitment time series that had a smaller magnitude than those models that fixed mature female M. As in previous years' assessments, no clear relationship existed between spawning biomass and recruitment. Also, regardless of the model used, the large recruitment event starting around 2014 appears to have persisted.

## Calculated OFLs and ABC

Medians of the posterior densities of the OFLs calculated for the suite presented models ranged from 24.91 to 29.92kt (Figure 16 & Table 4). Differences in OFLs were a result of differences in estimated MMB (see above), calculated  $B_{35\%}$  (which ranged from 139.9 to 150.19kt), Figure 16),  $F_{35\%}$  (which ranged from 1.28 to 1.36  $\text{yr}^{-1}$ , Figure 16), and  $F_{OFL}$  (which ranged from 0.88 to 0.93  $\text{yr}^{-1}$ , Figure 16).

Estimates of the distributions of management quantities from maximum likelihood-based standard errors and draws from the posterior distribution via an MCMC algorithm were similar, but not identical, for the 2017\_newgrowth model (Figure 16). The medians of the distribution of OFL and  $B_{35\%}$  were more similar across Bayesian and maximum likelihood methods than  $F_{35\%}$  and  $F_{OFL}$ . Uncertainty in the estimated time series of mature male biomass was also similar for Bayesian and maximum likelihood methodologies (Figure 17).

## Discussion

Models in which mature female natural mortality was estimated fit the data better than models in which mature female M was fixed. Estimating mature female M also eliminated the bimodality in management quantities and restored the proper relationship between estimated natural mortality for females and males. However, an increase of survey catchability for females to 1 was an unfortunate knock-on effect of this model change, which did not occur when natural mortality was fixed for mature females. Adding the additional growth data also removed the bimodality from estimates of management quantities resulting from fitting a model that fixed mature female M. Given the issue of increased estimated catchability relates primarily to females, and therefore will not impact the management quantities drastically, estimating natural mortality and incorporating the new growth data may be the most reasonable steps to improve the assessment for snow crab this year.

Estimating mature female M and incorporating the new growth data are recommended steps to improving the realism of the model, but these measures resulted in survey catchability of 1 for females. A revisitation of the treatment of the BSFRF survey data may help with this issue. Currently, an 'availability' curve is estimated (freely), but an empirical measure of availability could be generated by comparing the length composition

data from the BSFRF data to the associated NMFS data. Reconsidering the assumption of a 50/50 sex ratio in recruitment might also address the ‘runs’ of auto-correlation in the residuals of the fits to the survey mature biomass data. In general, the female mature biomass at the time of the survey is overestimated and male mature biomass at the time of the survey is underestimated (Figure 4). That said, little biological evidence (other than the runs in residuals from the assessment) exists to suggest the sex ratio in recruitment should deviate from 50/50. A possible hypothesis behind different sex ratios in recruitment is that initial sex ratios are 50/50 (i.e. the sex ratio at the time of fertilization), but mortality is size-based. Females may grow more slowly than males at smaller sizes (which is partially corroborated by the observed larger range of growth increments for males in the ~20mm size classes Figure 3). Consequently, females entering the model at 27.5 mm will have undergone more natural mortality than males because it takes them longer to grow to the size at which they recruit to the model, which would suggest a potentially different sex ratio at recruitment to the model. Differences in movement rates by sex from northern areas beyond the reach of the survey may also be a potential avenue to explore differences in sex ratio in recruitment.

The addition of the new growth data and the estimation of mature female  $M$  removed the bimodality in management quantities, which also removed the need to use MCMC to characterize posterior distributions of management quantities that spanned both modes. Bayesian methodologies can be a useful tool in assessment, but, when bimodality does not exist in model estimates, the differences in posterior distributions of management quantities compared to distributions generated by using the estimated standard errors were small for the snow crab assessment. The differences in these distributions were likely at least partially influenced by the assumed priors in the assessment. Given the time required to run MCMC and the time required to properly specify every prior in the model, maximum likelihood methods may be preferable to Bayesian methods for snow crab assessments performed in the near future. However, if circumstances arise in which management quantities are again bimodal, MCMC may again prove a useful tool for calculating quantities used in management under uncertainty.

Table 1: Parameter bounds and symbols

Parameter	Lower	Upper	Symbol	Process
af	-100	0	$\alpha_f$	growth
am	-50	0	$\alpha_m$	growth
bf	1	10	$\beta_{f,1}$	growth
bm	1	5	$\beta_{m,1}$	growth
b1	1	1.5	$\beta_{f,2}$	growth
bf1	1	2	$\beta_{m,2}$	growth
deltam	10	50	$\delta_m$	growth
deltaf	5	50	$\delta_f$	growth
st_gr	0.5	0.5	stgr	growth
growth_beta	0.749	0.751	$\beta_g$	growth
mateste	-6	-1e-10	$\Omega_{m,l}$	maturity
matestfe	-6	-1e-10	$\Omega_{f,l}$	maturity
mean_log_rec	“-inf”	Inf	$Rec_{avg}$	recruitment
rec_devf	-15	15	$Rec_{f,dev,y}$	recruitment
alpha1_rec	11.49	11.51	$\alpha_{rec}$	recruitment
beta_rec	3.99	4.01	$\beta_{rec}$	recruitment
mnatlen_styr	-3	15	$\lambda_{male,v,l}$	Initial N
fnatlen_styr	-10	15	$\lambda_{fem,v,l}$	Initial N
log_avg_fmort	“-inf”	Inf	$F_{avg,dir}^{log}$	Fishing mortality
fmort_dev	-5	5	$F_{dev,dir,y}^{log}$	Fishing mortality
log_avg_fmortdf	-8	-1e-04	$F_{avg,disc}^{log}$	Fishing mortality
fmortdf_dev	-15	15	$F_{dev,disc,y}^{log}$	Fishing mortality
log_avg_fmortt	-8	-1e-04	$F_{avg,trawl}^{log}$	Fishing mortality
fmortt_dev_era1	-15	15	$F_{dev,trawl,era1}^{log}$	Fishing mortality
fmortt_dev_era2	-15	15	$F_{dev,trawl,era2}^{log}$	Fishing mortality
log_avg_sel50_mn	4	5	$S_{50,new,dir}$	Fishing selectivity
log_avg_sel50_mo	4	5	$S_{50,old,dir}$	Fishing selectivity
fish_slope_mn	0.1	0.5	$S_{slope,m,d}$	Fishing selectivity
fish_fit_slope_mn	0.05	0.5	$S_{slope,m,d}$	Fishing selectivity
fish_fit_sel50_mn	85	120	$S_{50,old,dir}$	Fishing selectivity
fish_slope_mo2	1.9	2	$S_{slope,m,d}$	Fishing selectivity
fish_sel50_mo2	159	160	$S_{50,old,dir}$	Fishing selectivity
fish_slope_mn2	0.01	2	$S_{slope,m,d}$	Fishing selectivity
fish_sel50_mn2	100	160	$S_{50,old,dir}$	Fishing selectivity

Parameter	Lower	Upper	Symbol	Process
fish_disc_slope_f	0.1	0.7	$S_{slope,m,d}$	Fishing selectivity
fish_disc_sel50_f	1	5	$S_{50,old,dir}$	Fishing selectivity
fish_disc_slope_tf	0.01	0.3	$S_{slope,trawl}$	Fishing selectivity
fish_disc_sel50_tf	30	120	$S_{50,trawl}$	Fishing selectivity
srv1_q	0.2	1	$q_{m,era1,surv}$	Survey selectivity
srv1_q_f	0.2	1	$q_{f,era1,surv}$	Survey selectivity
srv1_sel95	30	150	$S_{95,era1,surv}$	Survey selectivity
srv1_sel50	0	150	$S_{50,era1,surv}$	Survey selectivity
srv2_q	0.2	1	$q_{m,era2,surv}$	Survey selectivity
srv2_q_f	0.2	1	$q_{f,era2,surv}$	Survey selectivity
srv2_sel95	50	160	$S_{95,era2,surv}$	Survey selectivity
srv2_sel50	0	80	$S_{50,era2,surv}$	Survey selectivity
srv3_q	0.2	1	$q_{m,era3,surv}$	Survey selectivity
srv3_sel95	40	200	$S_{95,m,era2,surv}$	Survey selectivity
srv3_sel50	25	90	$S_{50,m,era2,surv}$	Survey selectivity
srv3_q_f	0.2	1	$q_{f,era3,surv}$	Survey selectivity
srv3_sel95_f	40	150	$S_{95,f,era2,surv}$	Survey selectivity
srv3_sel50_f	0	90	$S_{50,f,era2,surv}$	Survey selectivity
srvind_q	0.1	1	$q_{m,09,ind}$	Survey selectivity
srvind_q_f	0.01	1	$q_{f,09,ind}$	Survey selectivity
srvind_sel95_f	55	120	$S_{95,f,09,ind}$	Survey selectivity
srvind_sel50_f	-50	55	$S_{50,f,09,ind}$	Survey selectivity
srv10in_q	0.1	1	$q_{m,10,ind}$	Survey selectivity
srv10ind_q_f	0.01	1	$q_{f,10,ind}$	Survey selectivity
selsmo10ind	-4	-0.001	SelVecMaleInd09	Survey selectivity
selsmo09ind	-4	-0.001	SelVecMaleInd10	Survey selectivity



Parameter	Lower	Upper	Symbol	Process
Mmult_imat	0.2	2	$\gamma_{natM,imm}$	Natural mortality
Mmult	0.2	2	$\gamma_{natM,mat,m}$	Natural mortality
Mmultf	0.2	2	$\gamma_{natM,mat,f}$	Natural mortality
cpueq	0.0000877	0.00877	$q_{cpue}$	CPUE q

Table 2: Estimated parameter values by scenario (these are maximum likelihood estimates)

Parameter	2016_oldgrowth	2016_newgrowth	2017_oldgrowth	2017_newgrowth
af	-4.96	-1.39	-5.26	0
am	-12.41	-0.92	-5.34	-0.85
bf	1.52	1.35	1.53	1.29
bm	1.84	1.37	1.52	1.36
b1	1.15	1.17	1.15	1.17
bfl	1.04	1.01	1.04	1.33
deltam	27.41	32.6	32.13	32.54
deltaf	34.31	41.29	34.13	27.45
mateste	vector	vector	vector	vector
matestfe	vector	vector	vector	vector
rec_devf	vector	vector	vector	vector
mnatlen_styr	vector	vector	vector	vector
fnatlen_styr	vector	vector	vector	vector
log_avg_fmort	-0.33	-0.3	-0.29	-0.24
fmort_dev	vector	vector	vector	vector
log_avg_fmortdf	-6.34	-6.28	-5.66	-5.89
fmortdf_dev	vector	vector	vector	vector
log_avg_fmortt	-4.82	-4.81	-4.61	-4.53
fmortt_dev_era1	vector	vector	vector	vector
fmortt_dev_era2	vector	vector	vector	vector
log_avg_sel50_mn	4.67	4.67	4.67	4.67
fish_slope_mn	0.19	0.19	0.19	0.19
fish_fit_slope_mn	0.42	0.43	0.43	0.43
fish_fit_sel50_mn	96.08	96.04	96.07	96.02
fish_disc_slope_f	0.24	0.25	0.25	0.26
fish_disc_sel50_f	4.26	4.26	4.25	4.23
fish_disc_slope_tf	0.09	0.09	0.07	0.07
fish_disc_sel50_tf	109.02	108.53	112.95	114.26
srv2_q	0.34	0.35	0.43	0.44
srv2_q_f	0.35	0.39	0.51	0.52
srv2_sel95	57.52	58.4	54.52	55.78
srv2_sel50	39.42	40.69	38.26	39.05
srv3_q	0.68	0.69	0.71	0.71
srv3_sel95	57.91	58.9	48.02	48.89
srv3_sel50	38.91	39.4	34.38	34.57
srv3_q_f	0.54	0.56	1	1
srv3_sel95_f	43.57	44.87	45.58	46.96
srv3_sel50_f	33.76	34.52	35.22	35.99
srvind_q	1	1	1	0.3
srvind_q_f	0.11	0.11	0.17	0.17
srvind_sel95_f	55	55	55	55
srvind_sel50_f	49.26	49.29	49.39	49.47
srv10ind_q_f	1	1	1	1
selsmo10ind	vector	vector	vector	vector
selsmo09ind	vector	vector	vector	vector
Mmult_imat	1.87	1.88	1.22	1.21
Mmult	1.07	1.07	1.16	1.16
Mmultf			1.55	1.52
cpueq	0	0	0	0

Table 3: Contribution to the objective function by individual likelihood component by modeling scenario. Note that some of the model scenarios involve changing the weightings of data sources or adding data which invalidate the direct comparison of likelihoods for a data source among models.

Likelihood component	2016_oldgrowth	2016_newgrowth	2017_oldgrowth	2017_newgrowth
Recruitment deviations	38.37	39.41	38.81	39.17
Initial numbers old shell males small length bins	5.14	5.07	4.73	4.71
ret fishery length	309.36	308.09	305.31	306.71
total fish length (ret + disc)	866.58	866.88	866.83	867.41
female fish length	236.3	237.66	233.89	233.65
survey length	4328.06	4316.68	4266.95	4329.5
trawl length	311.92	308.4	265.69	268
2009 BSFRF length	-86.59	-86.89	-93.56	-90.2
2009 NMFS study area length	-68.52	-69.19	-74.83	-72.74
M multiplier prior	18.33	18.68	81.53	73.62
maturity smooth	37.72	45.09	36.73	43.69
growth males	41.81	141.12	36.46	141.78
growth females	127.54	405.36	117.57	359.35
2009 BSFRF biomass	0.37	0.39	0.38	0.24
2009 NMFS study area biomass	0.09	0.1	0.12	0.21
cpue q	0.22	0.22	0.18	0.18
retained catch	3.8	3.92	3.88	3.94
discard catch	145.49	152.04	157.39	152.54
trawl catch	8.17	8	7.08	6.9
female discard catch	5.33	5.32	5.36	5.35
survey biomass	314.7	310.86	281.73	282.81
F penalty	25.13	25.31	24.64	25.3
2010 BSFRF	3.83	3.21	20.78	20.61
Biomass				
2010 NMFS	1.44	2.01	1.45	1.4
Biomass				
Extra weight survey lengths first year	564.67	564.95	553.32	551.4

Likelihood component	2016_oldgrowth	2016_newgrowth	2017_oldgrowth	2017_newgrowth
2010 BSFRF	-49.09	-51.5	-49.58	-47.24
length				
2010 NMFS	-55.91	-56.49	-58.37	-57.78
length				
smooth	2.45	2.96	2.99	1
selectivity				
smooth female	0	0	0	0
selectivity				
init nos smooth	47.49	48.07	45.81	45.85
constraint				
Total	7184.2	7555.73	7083.27	7497.36

Table 4: Changes in management quantities for each scenario considered. Reported management quantities are maximum likelihood estimates.

Model	MMB	B35	F35	FOFL	OFL
2016_oldgrowth	92.18	150.2	1.35	0.9	26.06
2016_newgrowth	89.66	147.2	1.33	0.88	24.91
2017_oldgrowth	96.97	140.5	1.28	0.88	29.92
2017_newgrowth	94.36	139.9	1.36	0.93	28.92

Table 5: Predicted mature male (MMB), mature female (FMB), and males >101mm biomass (1000 t) and numbers (in millions) at the time of the survey from the chosen model. Columns 2-5 are subject to survey selectivity; columns 6-9 are the population values (e.g. they are not modified by multiplying them by a selectivity curve—they are estimates of the underlying population). These are maximum likelihood estimates that differ slightly from median posterior values.

Survey year	FMB	MMB	Male >101 biomass	Male >101 (millions)	FMB	MMB	Male >101 biomass	Male >101 (millions)
1982	65.29	124.4	34.21	65.04	133.8	282.5	57.02	108.4
1983	53.95	131.9	57.64	103.2	109.7	299.8	96.06	171.9
1984	41.12	139.1	78.74	135.5	83.72	316.3	131.2	225.8
1985	40.6	132.9	81.51	138	83.15	302.4	135.8	230
1986	51.31	116.8	48.89	82.54	105.5	266.2	110.8	187.1
1987	85.6	111.7	41.27	71.14	176.6	255.3	93.58	161.3
1988	207.1	188.3	36.48	63.24	210.3	265.3	82.71	143.4
1989	235.4	219.7	40.93	72.49	239.3	309.5	92.8	164.4
1990	218.4	284.4	69.48	121.5	221.6	400.4	157.5	275.4
1991	175.9	270.7	66.75	115.5	178.4	381	151.3	261.9
1992	140.4	226.6	53.45	93	142.4	319	121.2	210.8
1993	187.9	193.7	74.3	125.9	191.3	272.9	104.4	176.9
1994	216.9	165	44.8	74.8	220.4	232.7	62.93	105.1
1995	197.6	182.5	42.9	75.83	200.5	257.2	60.27	106.5
1996	156.7	257.3	103.3	181.4	158.9	362.1	145.2	254.8
1997	117	308.9	166.3	279.6	118.6	434.4	233.6	392.7
1998	86.88	234.2	119.6	198.7	88.08	329.5	168	279.1
1999	74.39	150	62.27	104.6	75.51	211	87.48	147
2000	72.47	121	48	80.16	73.61	170.3	67.44	112.6
2001	65.72	102	36.71	62.01	66.69	143.6	51.57	87.11
2002	55.32	95.1	34.35	59.33	56.12	133.9	48.26	83.35
2003	50.43	99.41	43.63	74.38	51.2	139.9	61.29	104.5
2004	58.9	99.7	48.27	80.47	59.89	140.3	67.8	113
2005	77.47	94.91	42.91	71.27	78.8	133.7	60.28	100.1
2006	88.02	96.46	37.93	64.41	89.43	135.9	53.28	90.49
2007	87.2	115.5	47.95	82.49	88.53	162.7	67.36	115.9
2008	74.77	134.9	63.44	108.6	75.85	189.9	89.13	152.5
2009	59.76	146.8	78.04	131.1	60.61	206.6	109.6	184.2
2010	60.49	142.5	80.08	132.8	61.48	200.5	112.5	186.6
2011	64.95	122.8	66.22	109.3	65.98	172.8	93.02	153.5
2012	63.26	91.32	37.52	63.56	64.23	128.6	52.71	89.29
2013	61.59	83.71	30.41	53.43	62.55	117.9	42.72	75.06
2014	61.45	89.29	37.46	64.36	62.42	125.7	52.62	90.42
2015	59.34	83.59	33.86	57.59	60.25	117.7	47.56	80.9
2016	75.52	87.45	36.14	61.38	76.83	123.2	50.78	86.22
2017	141.6	107.7	45.45	76.38	144.3	151.9	63.84	107.3

Table 6: Maximum likelihood estimates of predicted mature male biomass at mating, mature female biomass at mating (in 1000 t), and recruitment (millions) from the chosen model. These are maximum likelihood estimates that differ slightly from the median posterior values.

Survey year	Mature male biomass	Mature female biomass	Recruits
1982	225.7	107.6	266.1
1983	240.6	88.16	1006
1984	236.5	67.28	1526
1985	210	66.83	3753
1986	176.4	84.72	1040
1987	153.2	141.9	2876
1988	155.3	169.1	96.91
1989	189.8	192.3	337.6
1990	193.8	178.1	484.4
1991	180.7	143.3	4152
1992	166.2	114.3	978.9
1993	161.8	153.7	540.4
1994	159.5	176.9	129.9
1995	187.4	161.1	78.67
1996	253.2	127.7	119.2
1997	253.1	95.32	564.8
1998	188.5	70.79	562.4
1999	162.6	60.68	175.1
2000	131.8	59.16	177.4
2001	105.8	53.6	395.7
2002	100.1	45.1	851.8
2003	107.1	41.14	1093
2004	106.7	48.13	653.5
2005	95.79	63.34	502.6
2006	97.91	71.86	106
2007	109.2	71.15	142.5
2008	134.3	60.96	801.5
2009	152.7	48.71	486.3
2010	144.7	49.41	329.9
2011	105	52.93	483.1
2012	77.78	51.62	461.7
2013	75.02	50.26	345
2014	75.29	50.06	1352
2015	80.66	48.41	3421
2016	94.36	61.75	2079

Table 7: Maximum likelihood estimates of predicted total numbers (millions), not subject to survey selectivity at the time of the survey. These are maximum likelihood estimates that differ slightly from the median posterior values.

Survey year	Total numbers
1982	5.956
1983	4.907
1984	5.623
1985	7.197
1986	12.82
1987	11.62
1988	14.29
1989	10.7
1990	8.421
1991	6.847
1992	13.04
1993	11.51
1994	9.516
1995	7.22
1996	5.405
1997	4.111
1998	3.967
1999	3.922
2000	3.242
2001	2.736
2002	2.789
2003	3.753
2004	4.972
2005	5.008
2006	4.708
2007	3.678
2008	2.951
2009	3.737
2010	3.724
2011	3.396
2012	3.423
2013	3.414
2014	3.18
2015	5.007
2016	10.56
2017	12.07



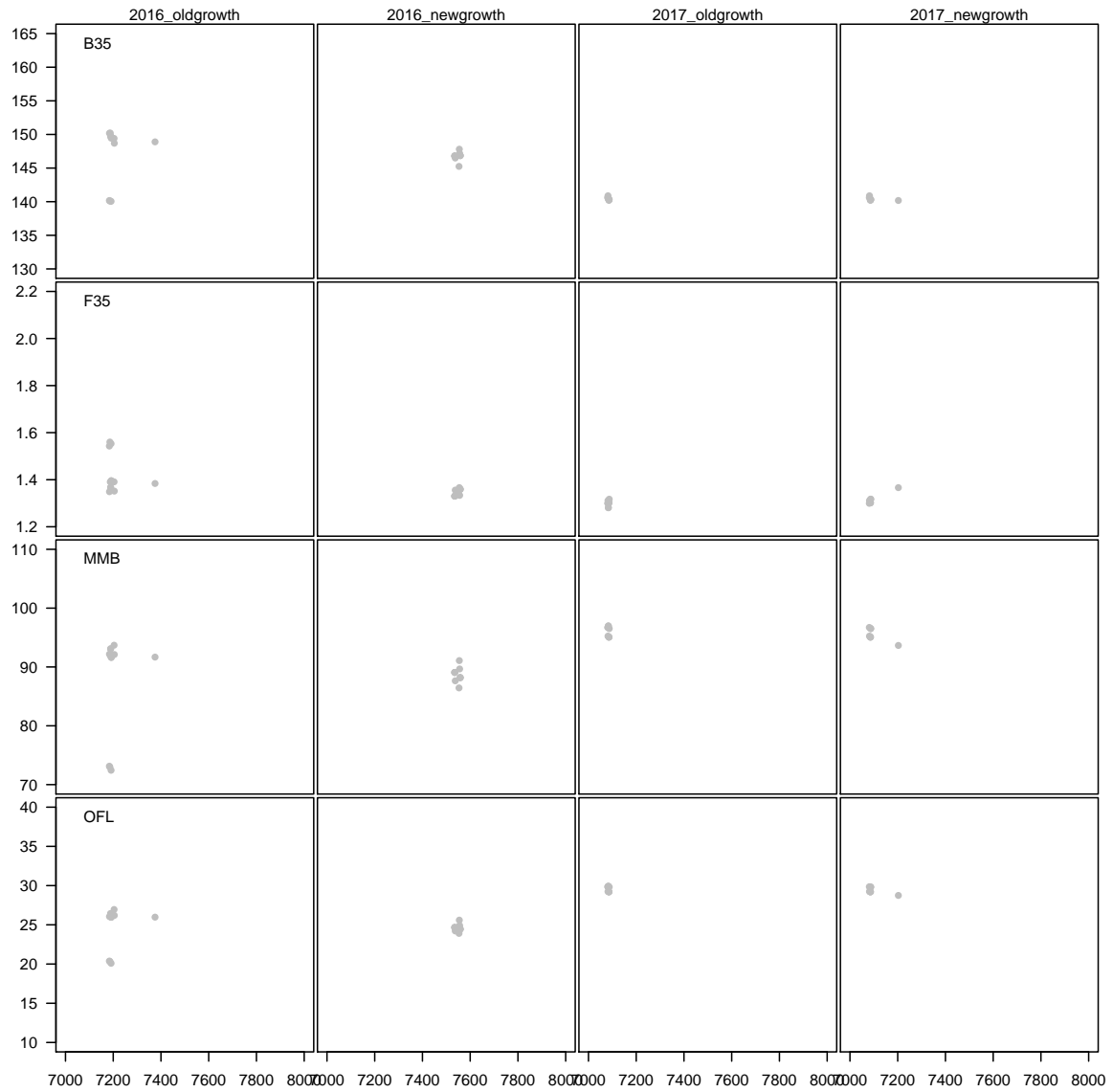


Figure 1: Management quantities after jittering all models. Each grey dot represents a model run—some grey dots are actually many model runs overlaid on one another.

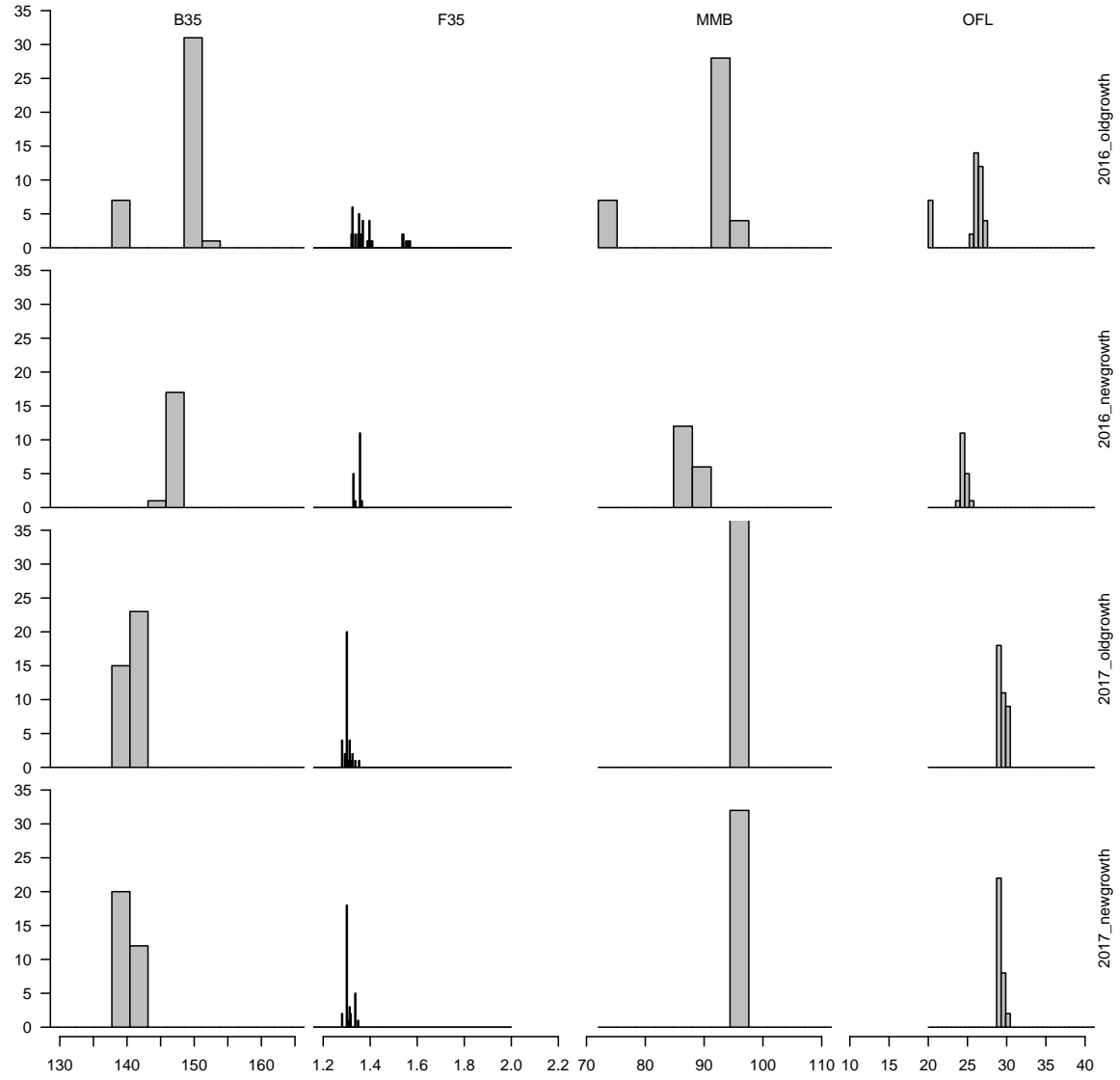


Figure 2: Histograms of management quantities after jittering all models.

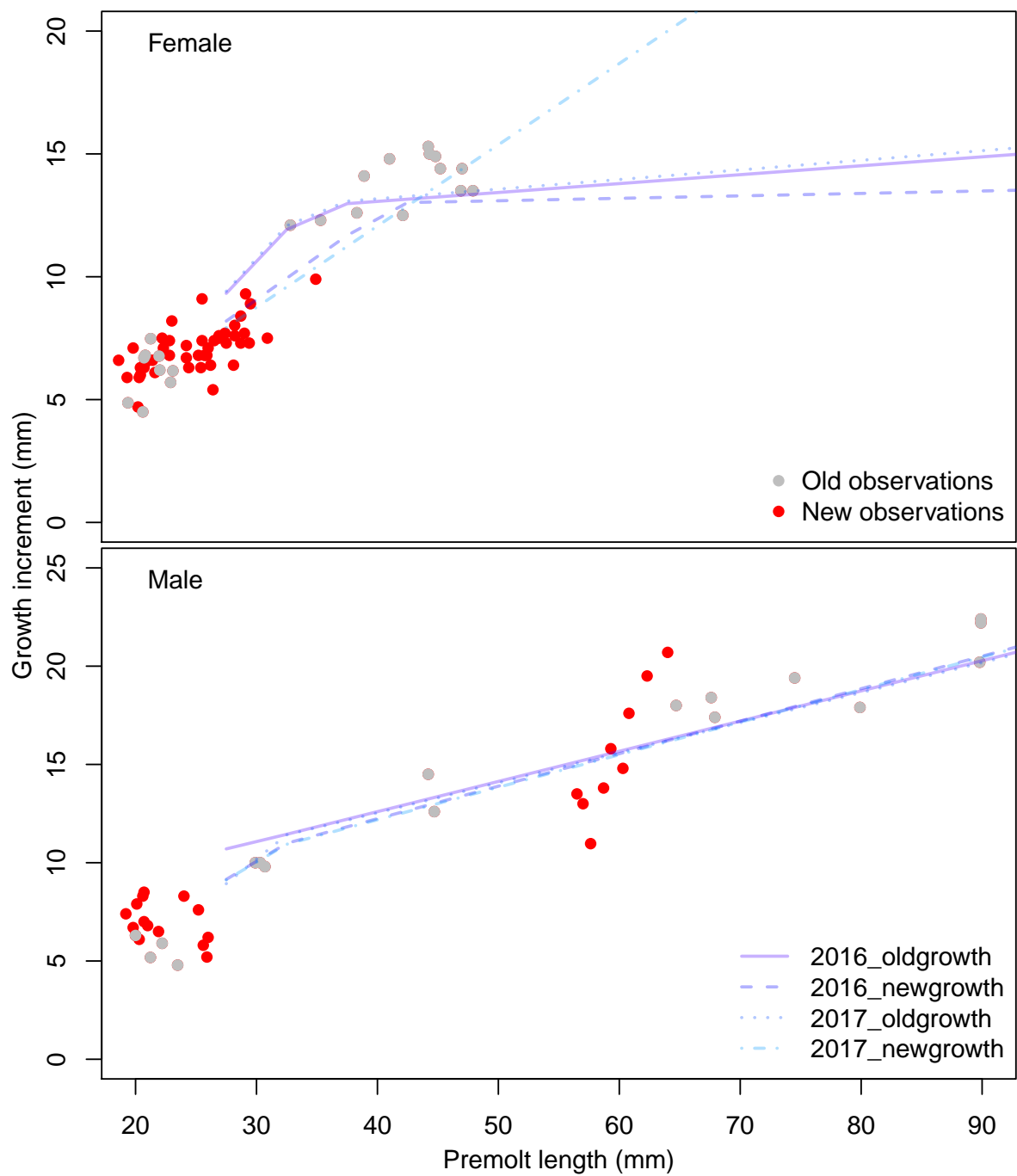


Figure 3: Model fits to the growth data

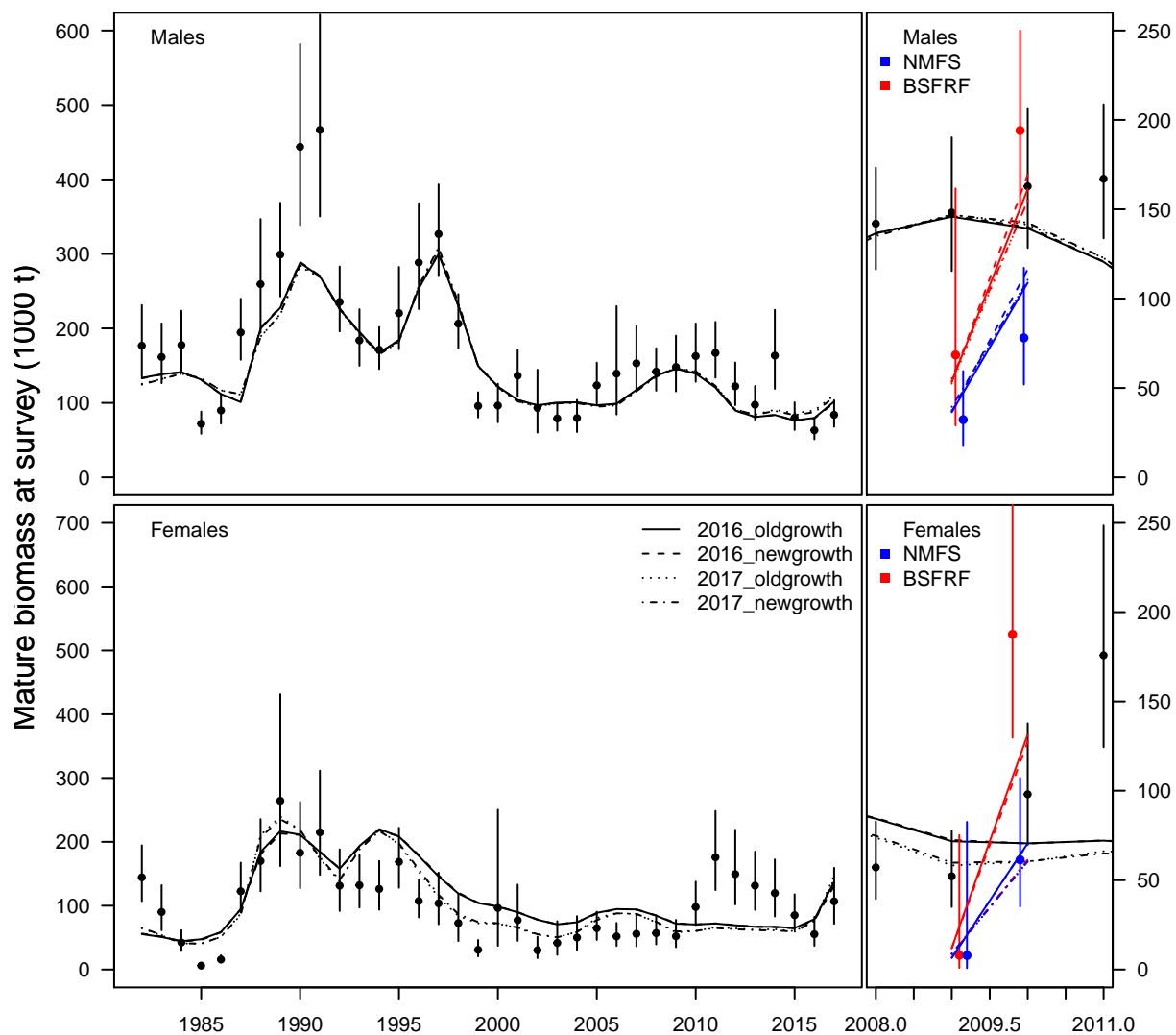


Figure 4: Model fits to the observed mature biomass at survey. Right column displays the fits to the biomass calculated from the BSFRF and NMFS selectivity experiments in 2009 and 2010. The biomass estimates from the experimental surveys are overlaid on the data from the NMFS trawl survey and model estimated trends.

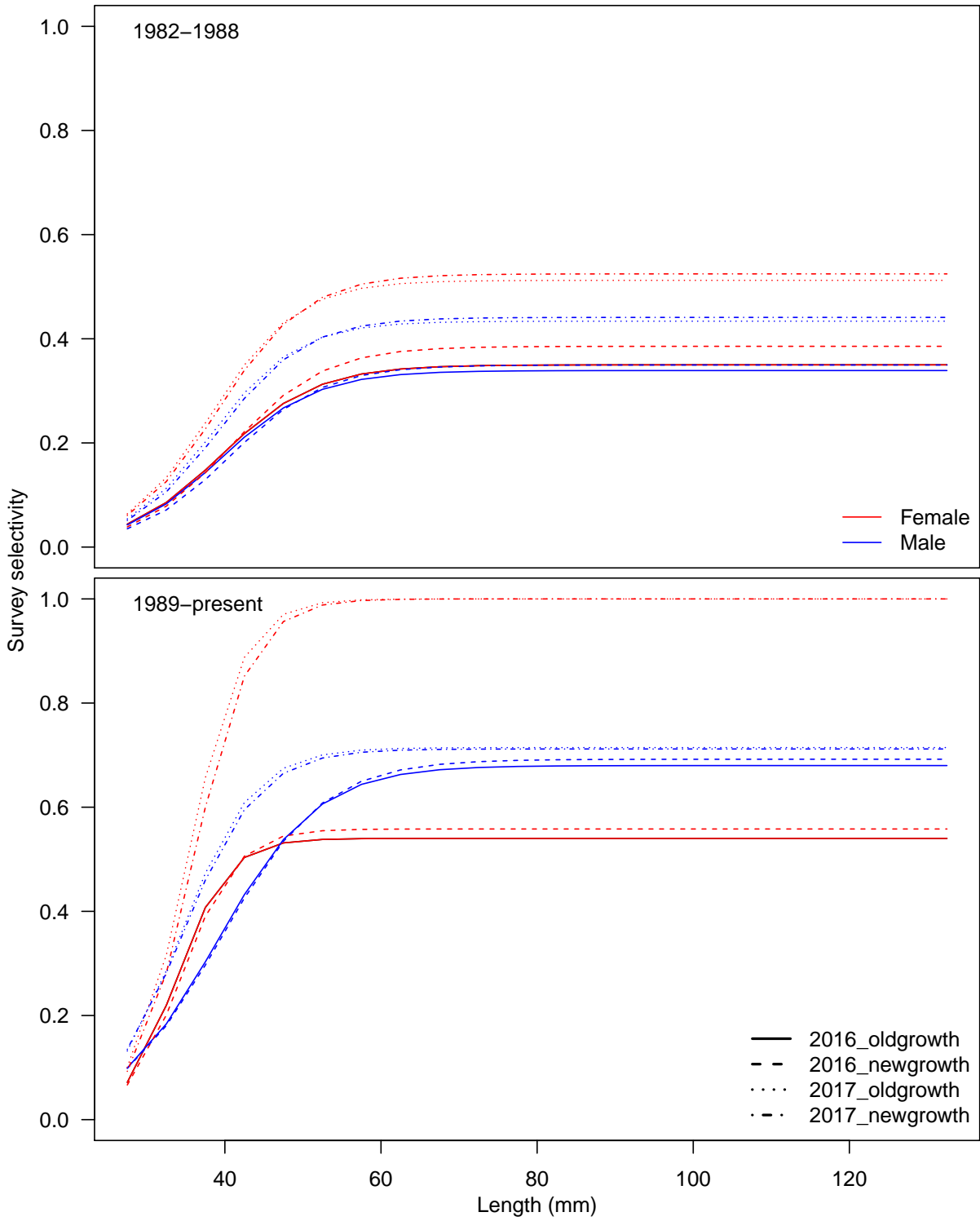


Figure 5: Estimated survey selectivity

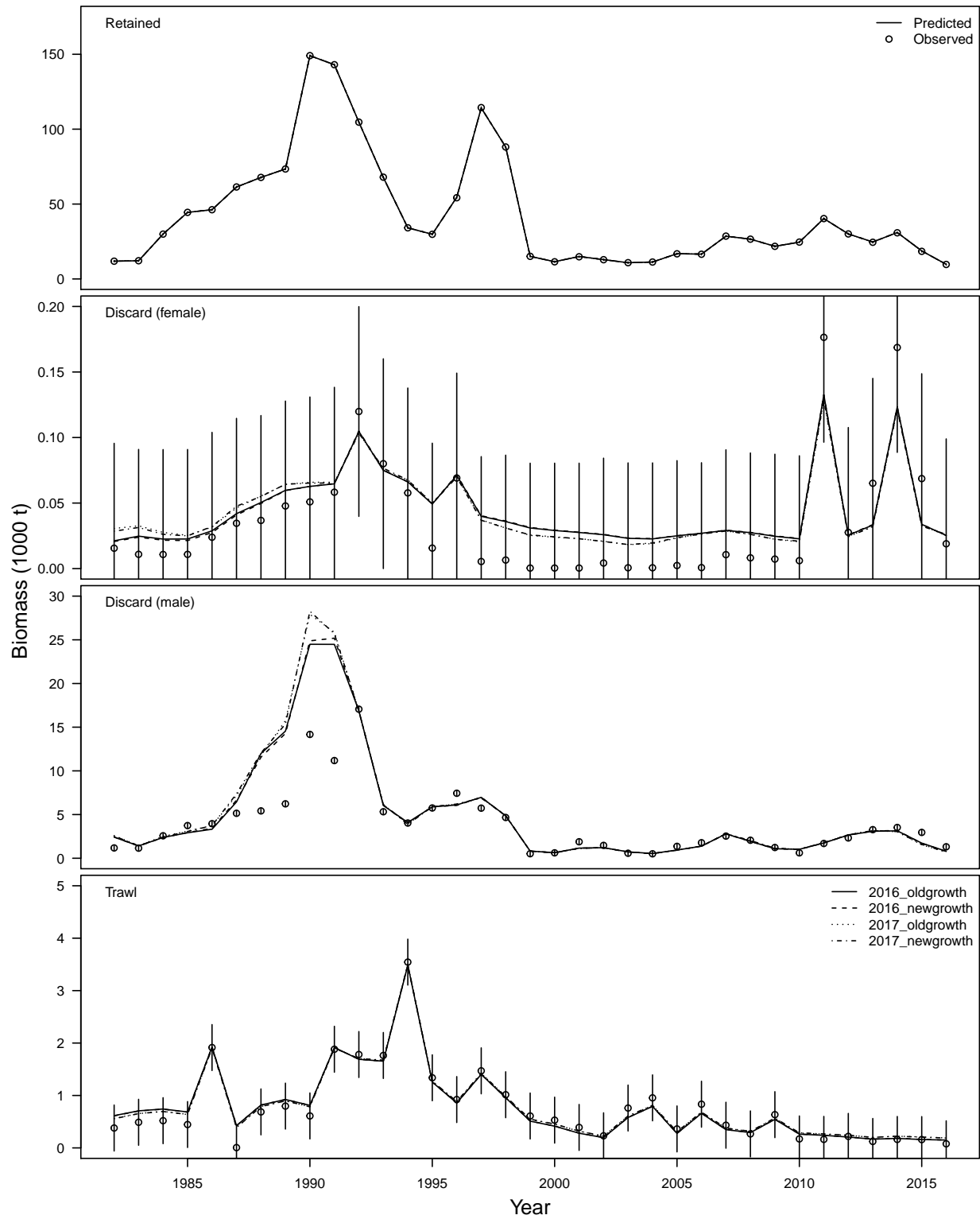


Figure 6: Model fits to catch data

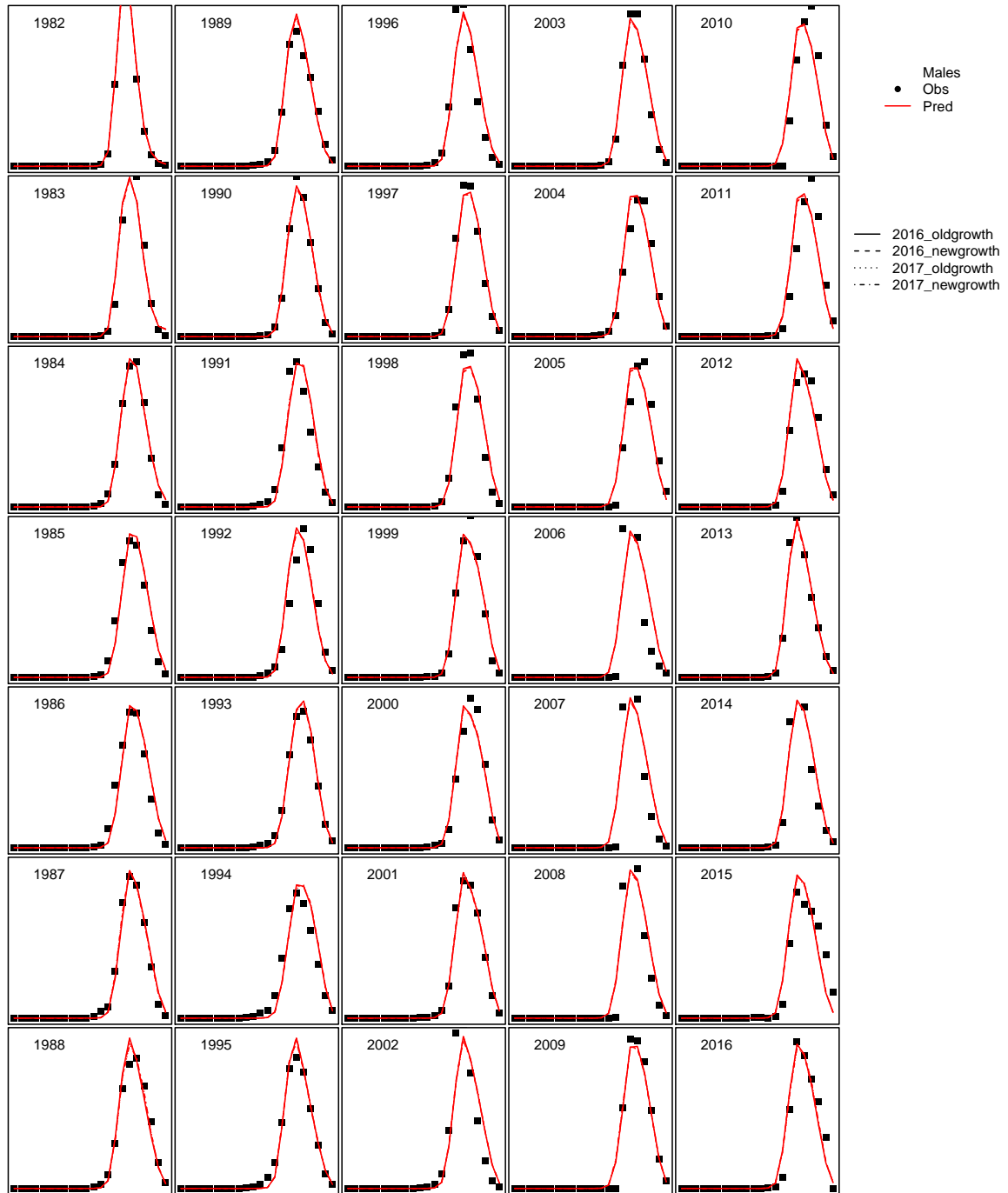


Figure 7: Model fits to retained catch size composition data

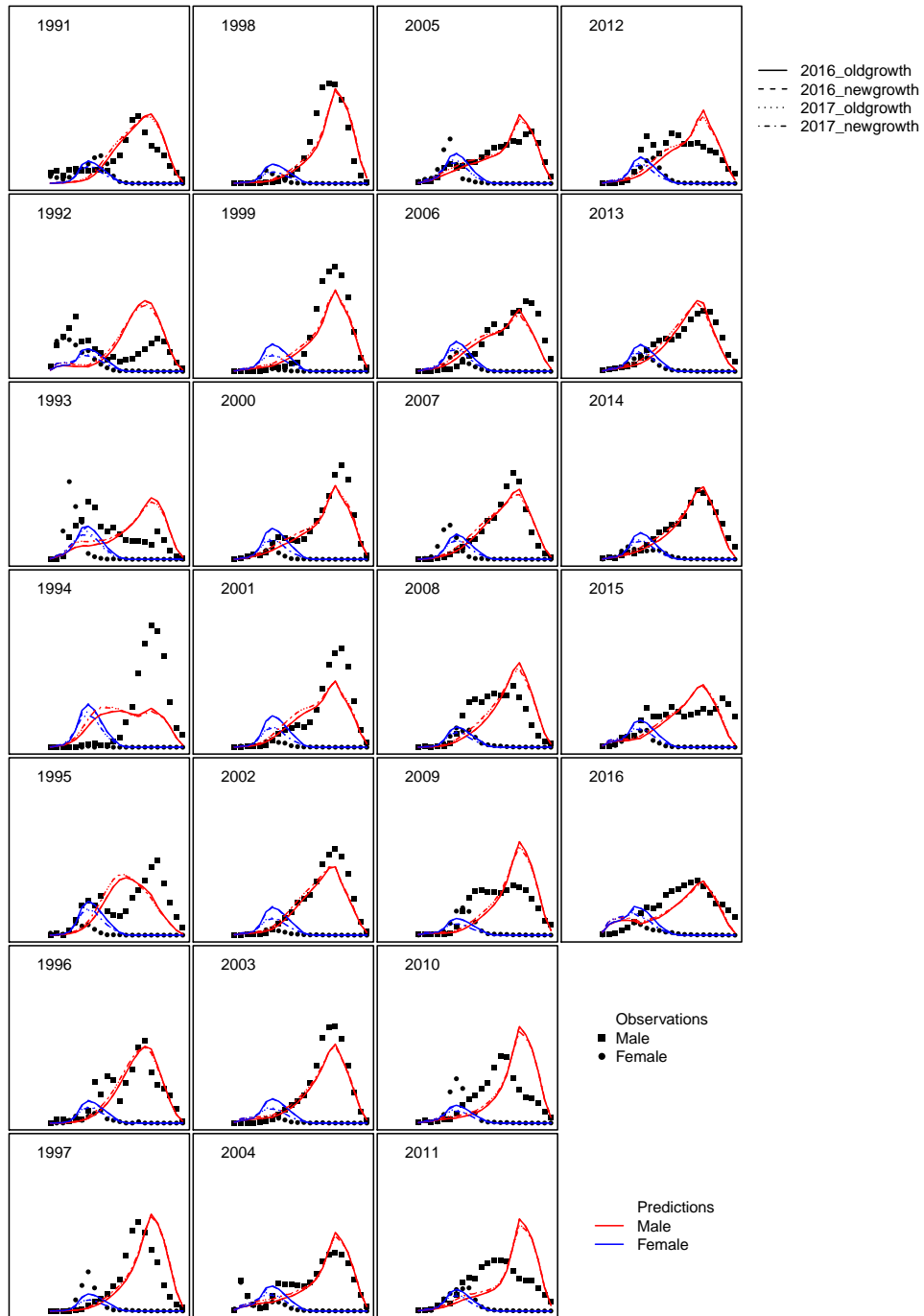


Figure 8: Model fits to trawl catch size composition data



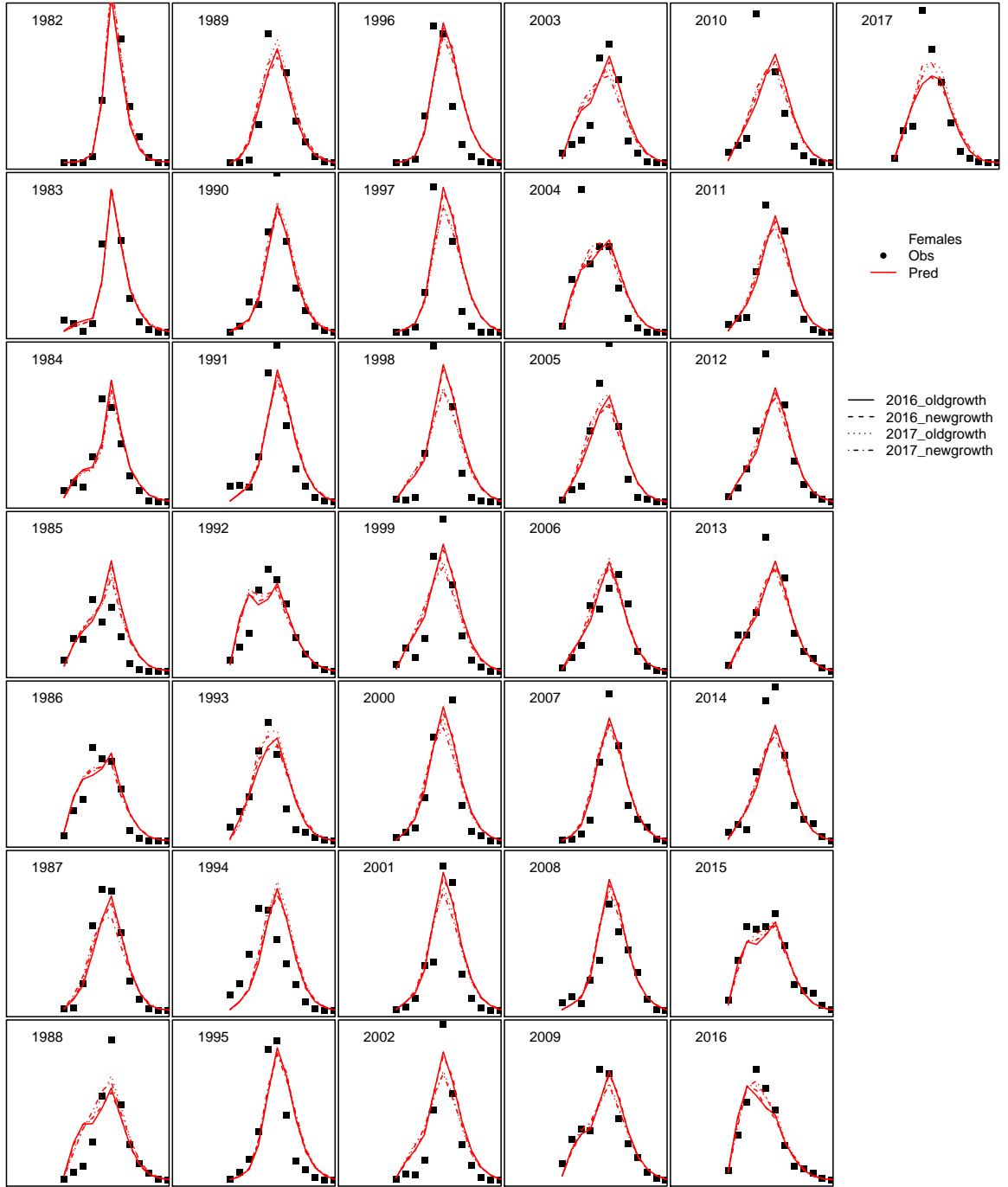


Figure 9: Model fits to female survey size composition data. Note that male and female survey selectivity proportions at length in a given year sum to 1. Consequently, the integral of predicted length compositions may appear to be different than the integral of the observed length composition data.

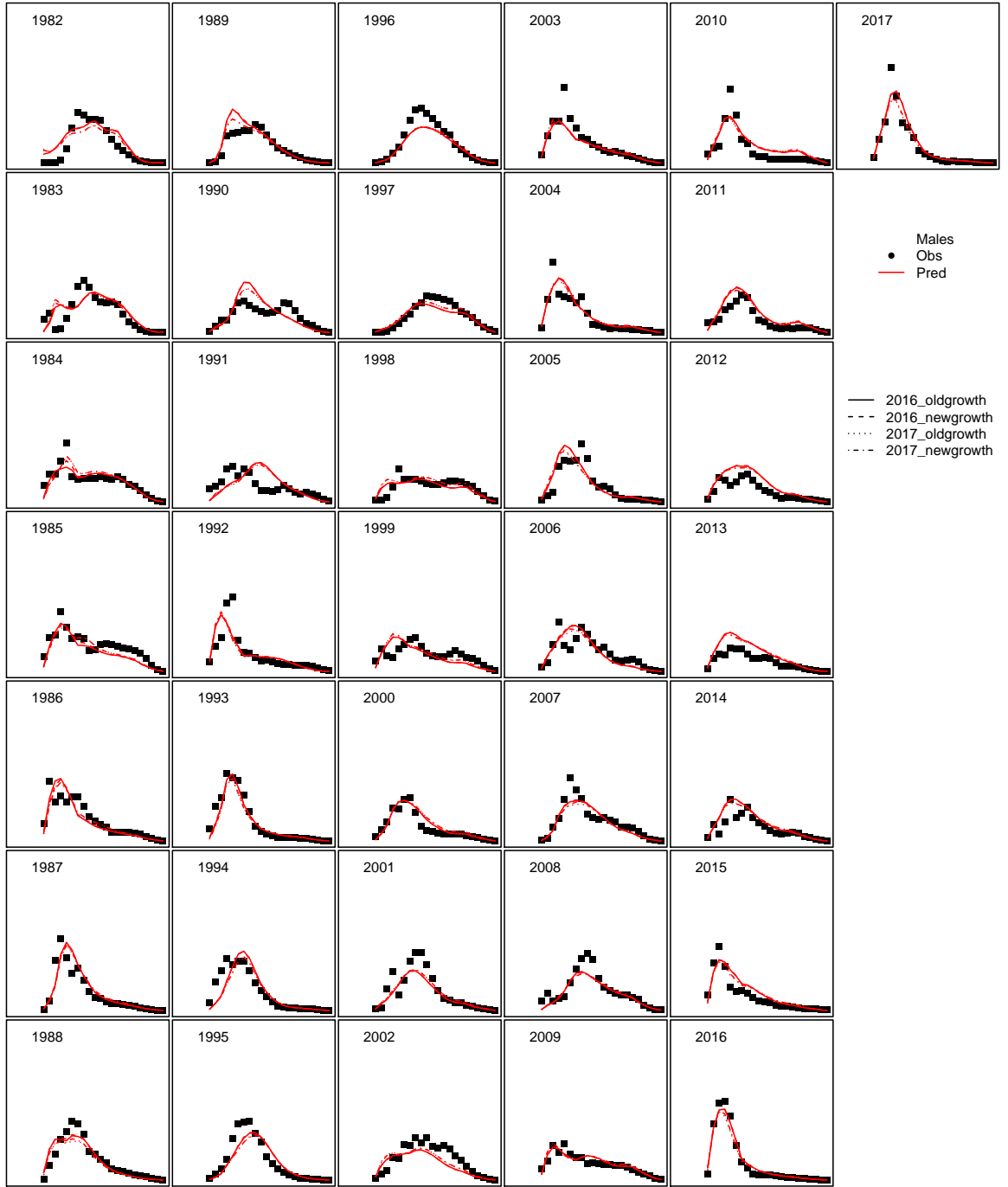


Figure 10: Model fits to male survey size composition data. Note that male and female survey selectivity proportions at length in a given year sum to 1. Consequently, the integral of predicted length compositions may appear to be different than the integral of the observed length composition data.

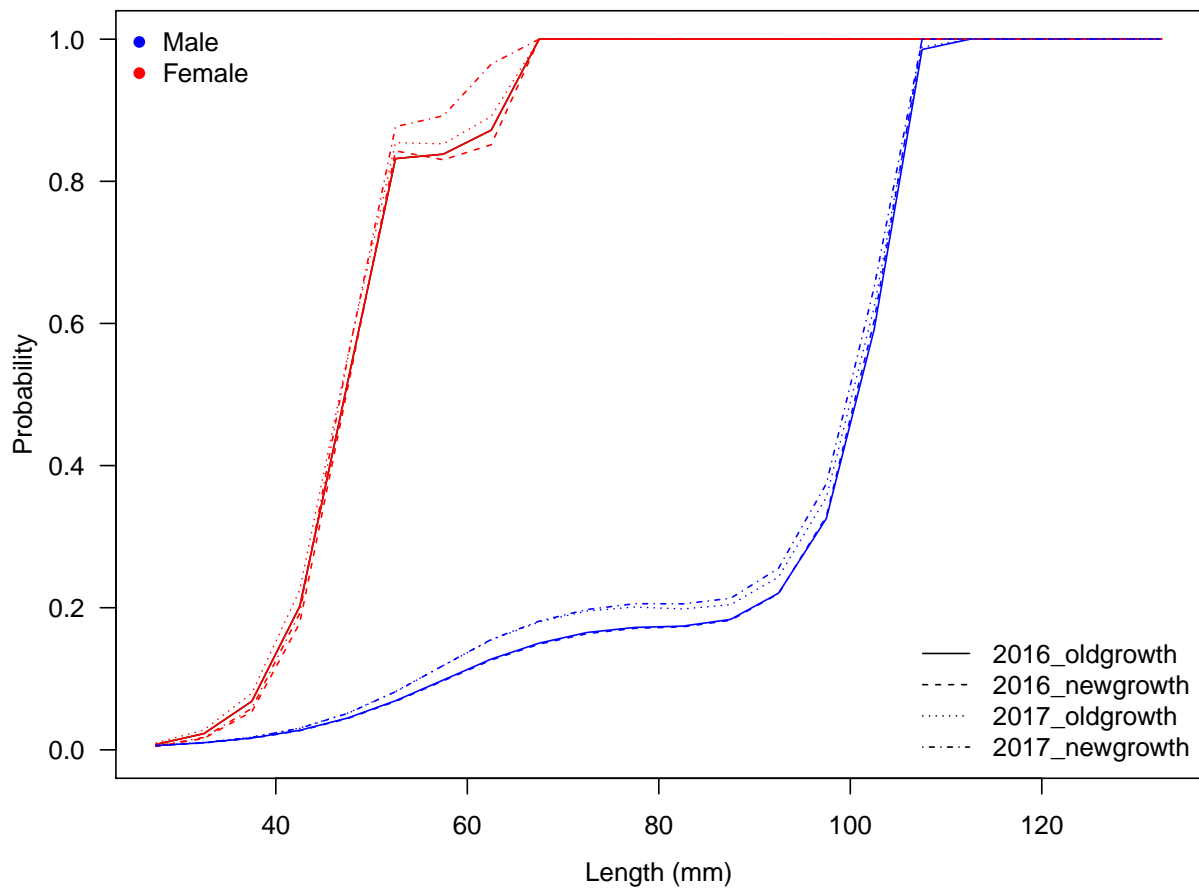


Figure 11: Estimated probability of maturing

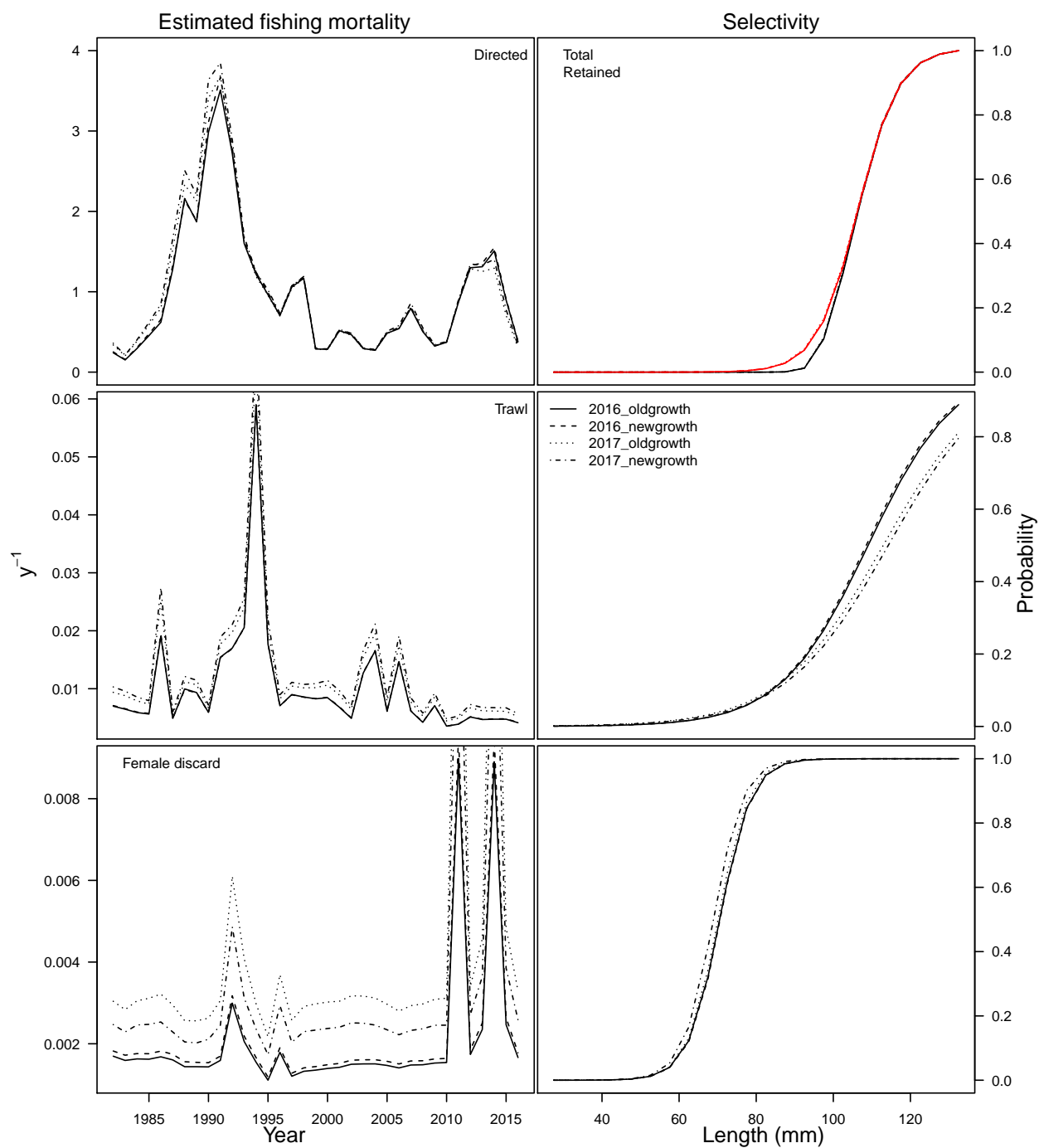


Figure 12: Model predicted fishing mortalities and selectivities for all sources of mortality

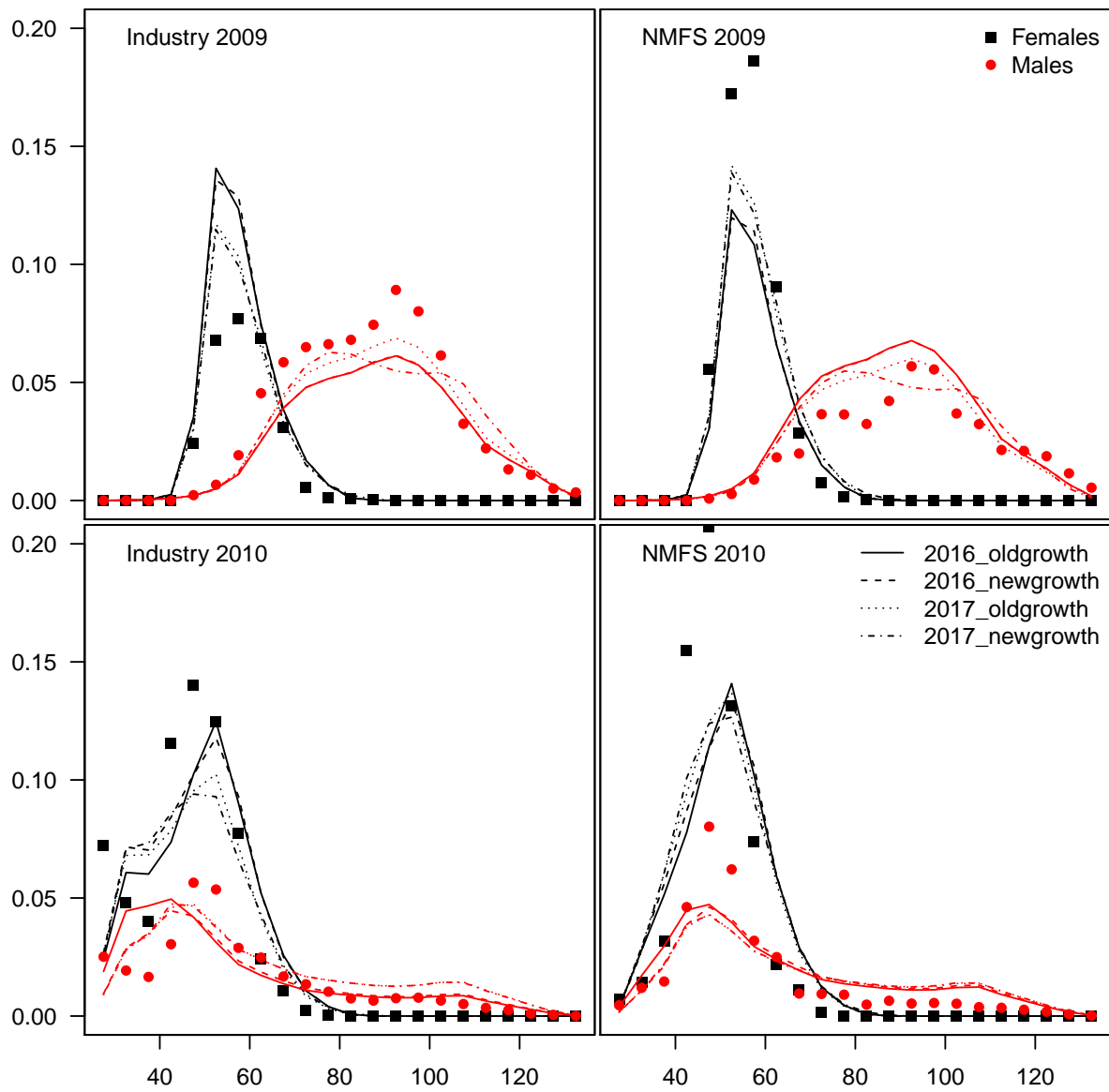


Figure 13: Model fits to size composition data from summer survey experiments (2009 & 2010)

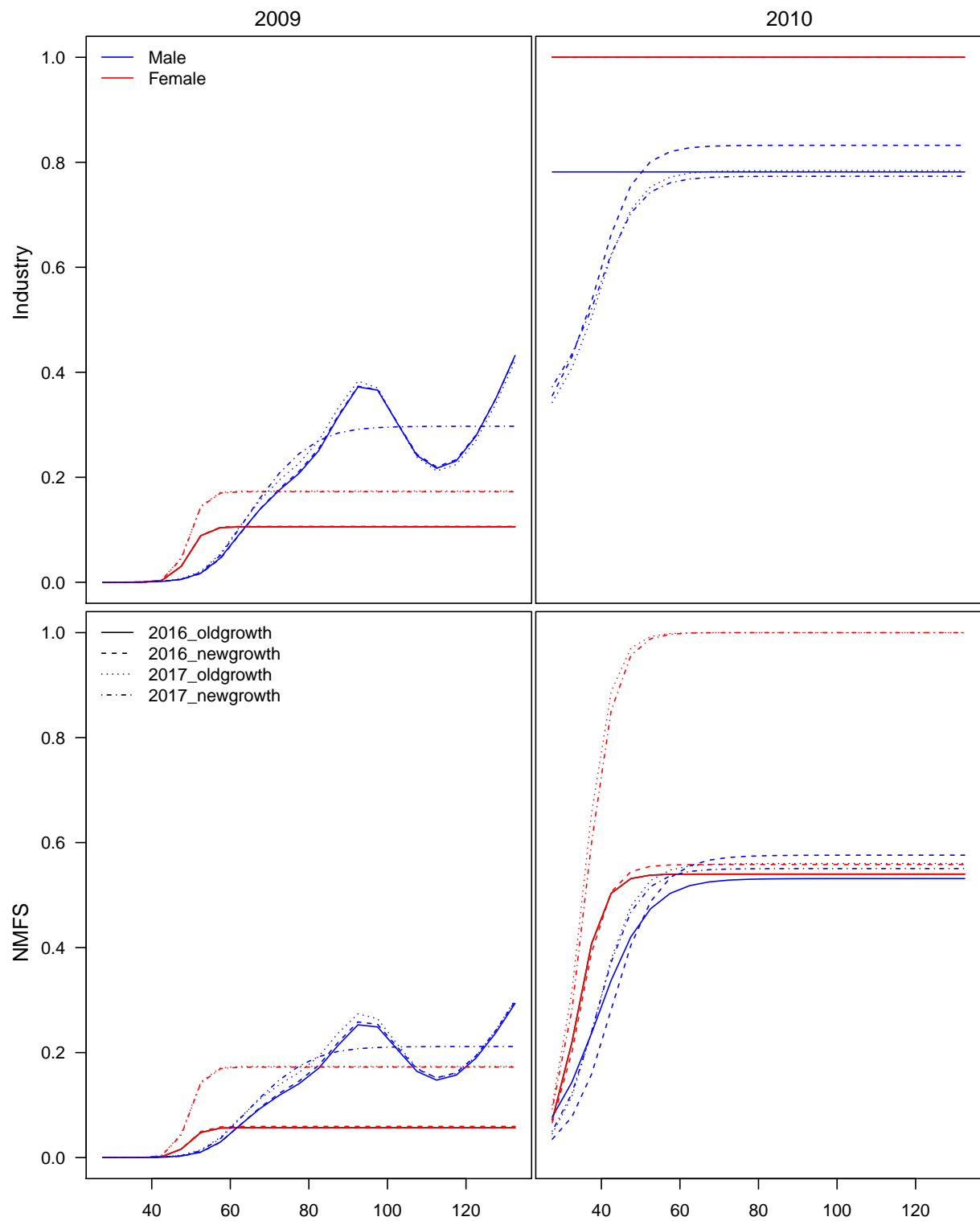


Figure 14: Estimated experimental survey selectivity (availability \* survey selectivity)

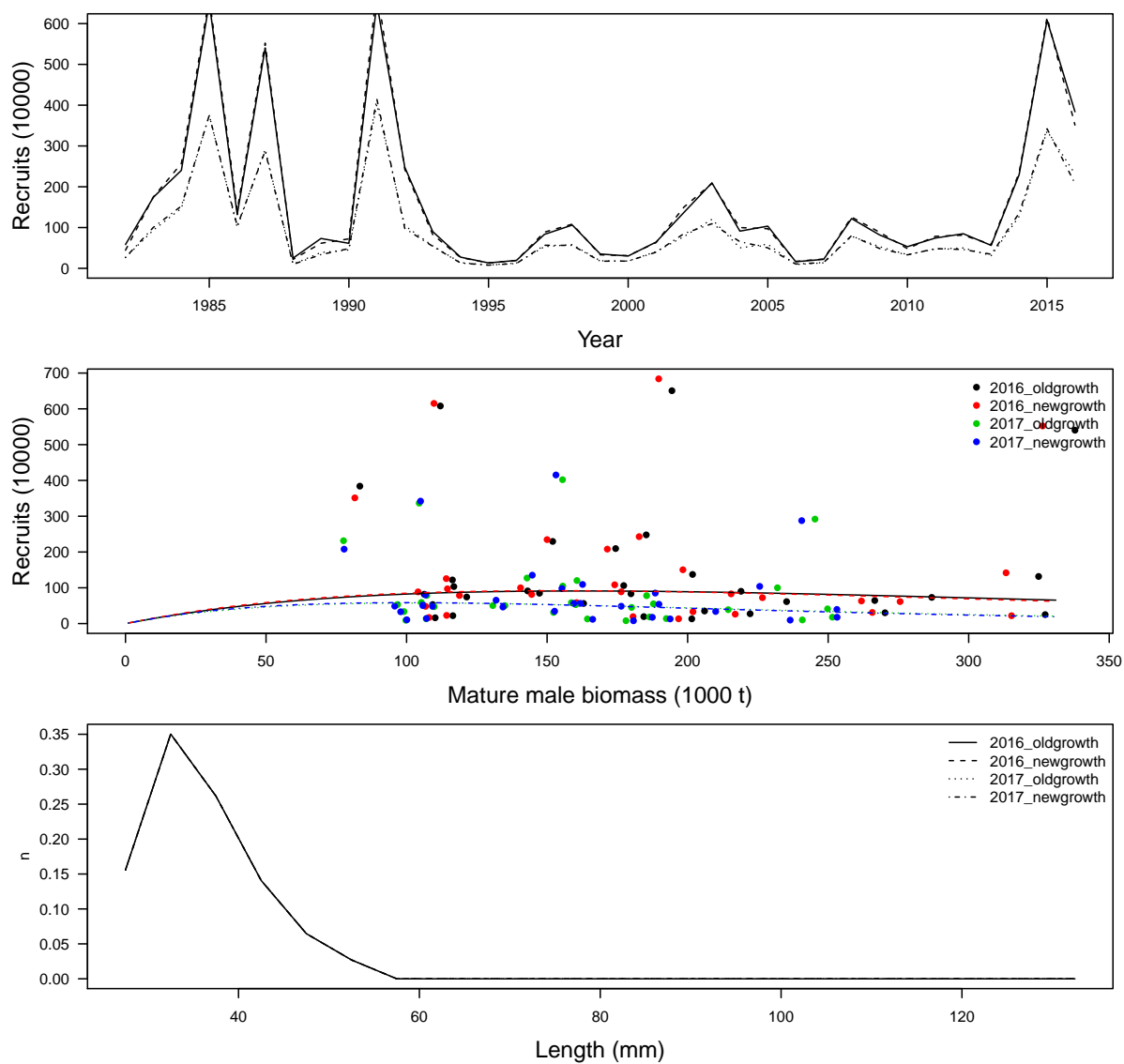


Figure 15: Estimated recruitment, fits to stock recruit curve (MMB lagged 5 years), and proportions recruiting to length bin

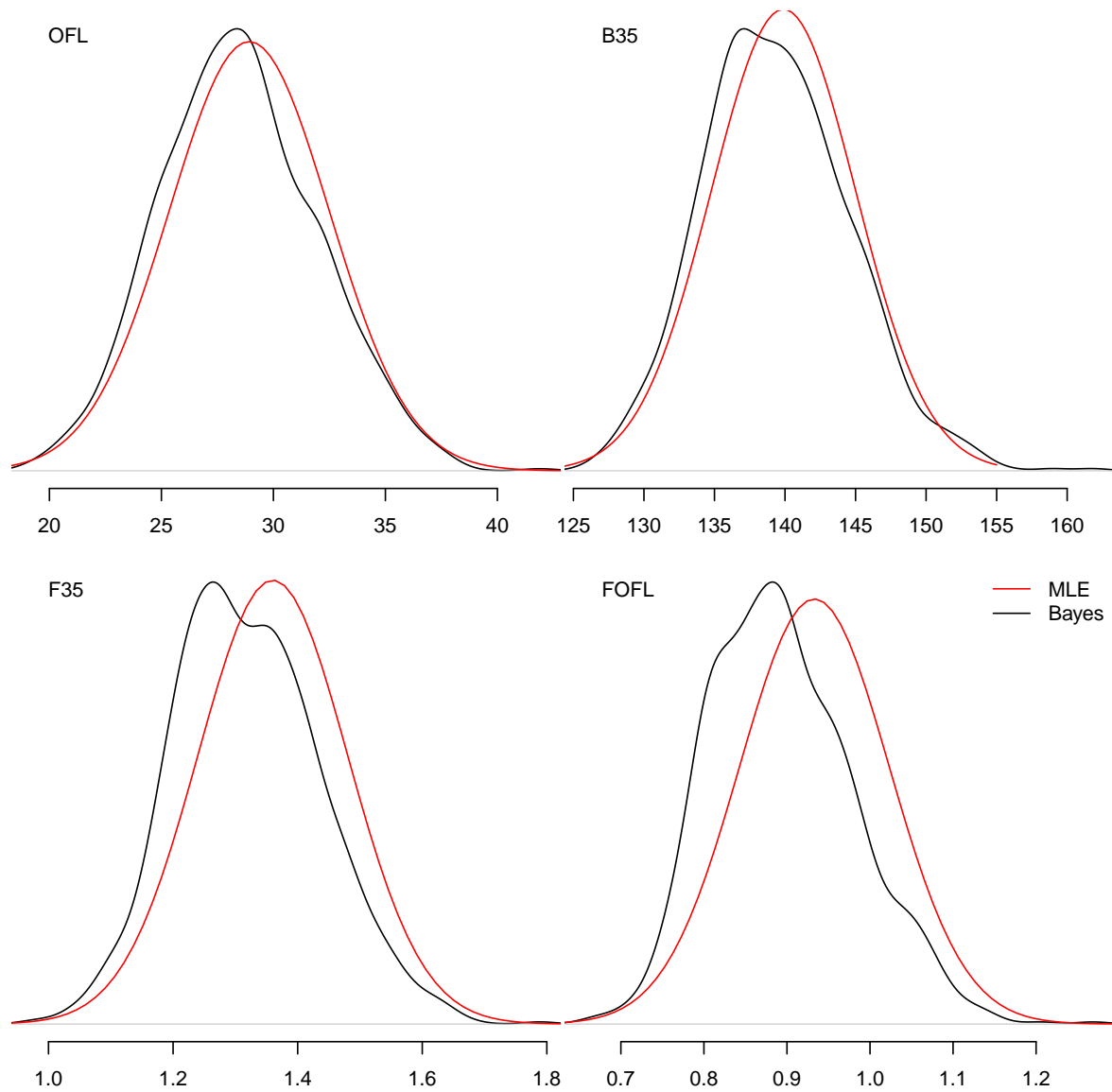


Figure 16: Posterior densities for management quantities by scenario compared to normal distributions generated using the standard errors estimated via maximum likelihood.



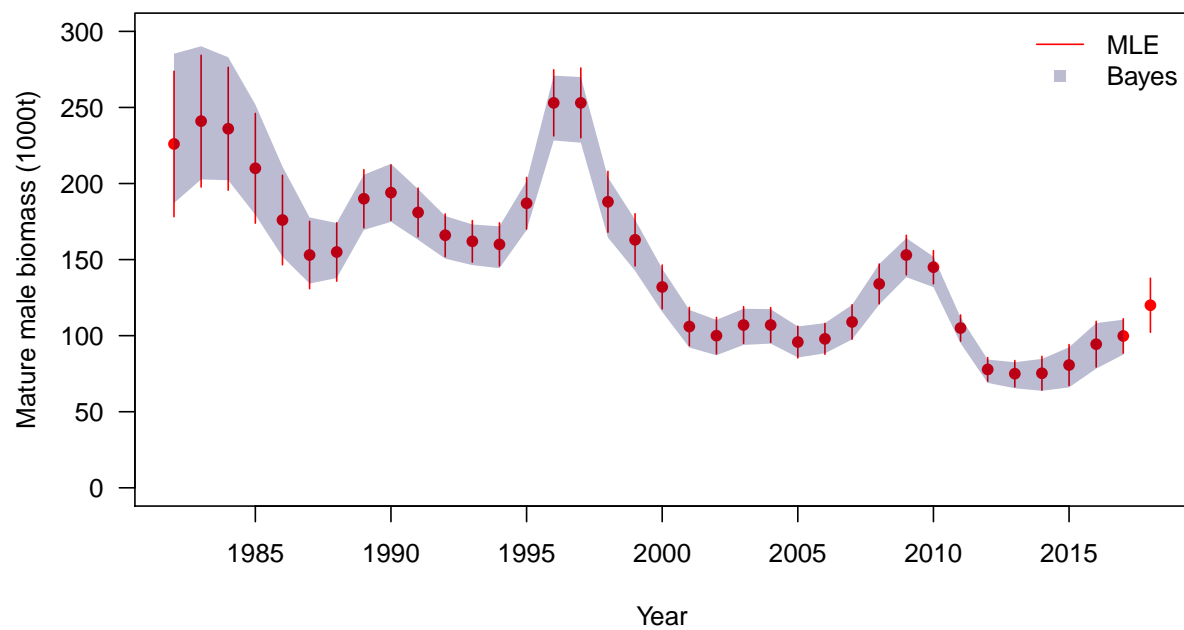


Figure 17: Comparison of MLE and Bayesian estimates of uncertainty in MMB